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Entanglement Measures from Quantum Field Theory Methods in 1+1 Dimensions

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SISSA, Trieste
28 April 2016

Related Papers:

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- This talk is mainly based on the papers:

  Olivier Blondeau-Fournier, O. C.-A. and Benjamin Doyon, *Universal scaling of the logarithmic negativity in massive quantum field theory*, J.Phys. A49 (2016) 125401. arXiv:1508.04026

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  Davide Bianchini, Olivier Blondeau-Fournier, O. C.-A. and Benjamin Doyon, *Entanglement Measures in the 1+1 Dimensional Massive Free Boson Theory*, in preparation.

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- What provides a good measure of entanglement? [Plenio & Virmani'05]
- The bi-partite entanglement entropy [Bennett et al.'96] and the logarithmic negativity [Peres'96; Eisert'00; Vidal & Werner'01; Plenio'05] are good measures of entanglement according to these properties

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$$\mathcal{E} = \log \mathrm{Tr}_{A\cup B}|\rho_{A\cup B}^{T_B}| \quad \text{with} \quad \rho_{A\cup B} = \mathrm{Tr}_C(|\Psi\rangle\langle\Psi|)$$

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- $|\Psi\rangle$ is the state of the whole system (for pure states)

Logarithmic Negativity (LN)

- The LN provides a good measure of entanglement in pure and mixed states for non-complementary regions such as A and B [Peres'96; Eisert'00; Vidal & Werner'01; Plenio'05]

- There is also a "replica" approach to the computation of the negativity [Calabrese, Cardy & Tonni'12]:

Logarithmic Negativity from the Replica Trick

$$\mathcal{E}[n] = \log \mathrm{Tr}_{A\cup B}(\rho_{A\cup B}^{T_B})^n \quad \text{then} \quad \mathcal{E} = \lim_{n\to 1} \mathcal{E}_e[n]$$

where $\mathcal{E}_e[n]$ means the function $\mathcal{E}[n]$ for n even. This limit requires analytic continuation from n even to $n = 1$

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$$\mathcal{E}^{\perp}(\ell_1, \ell_2) \sim \frac{c}{4} \log \frac{\ell_1 \ell_2}{\ell_1 + \ell_2}$$

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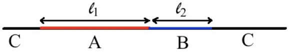

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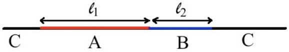

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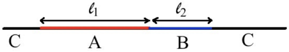

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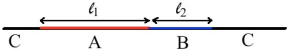

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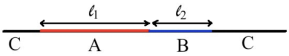

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Universal saturation and decay: For adjacent regions ($\ell_1 := \ell$, $\ell_2 \to \infty$) [Blondeau-Fournier, OC-A & Doyon'15]

$$\mathcal{E}^{\perp}(\ell) \sim -\frac{c}{4}\log(m\epsilon) + \mathcal{E}_{\rm sat} - \frac{2a}{3\sqrt{3}\pi} K_0(\sqrt{3}m\ell)$$

where $m \propto \xi^{-1}$ is the smallest mass scale in the theory

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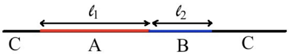

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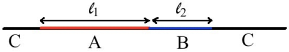

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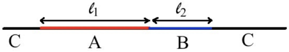

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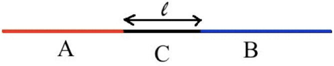
For semi-infinite non-adjacent regions:



$$\mathcal{E}^{\dashv\vdash}(\ell) \sim \frac{a(m\ell)^2}{2\pi^2}\left[K_0(m\ell)^2 + \frac{K_0(m\ell)K_1(m\ell)}{m\ell} - K_1(m\ell)^2\right]$$

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- We later proposed an interpretation of this field as a branch point twist field and characterize it through its locality properties versus other fields in the replica theory [Cardy, O.C-A & Doyon'08]

QFT Definition of Twist Fields

- The Twist Fields are defined through very general conmutation relations with the fundamental field of the model [Cardy, O.C-A & Doyon'08]:

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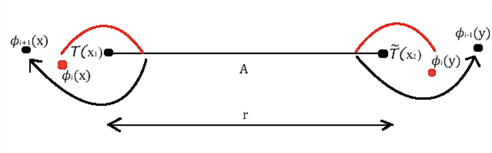
- Diagramatically:



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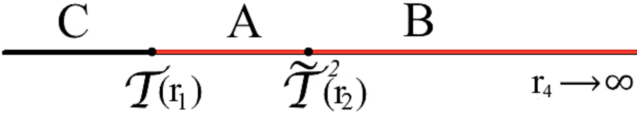

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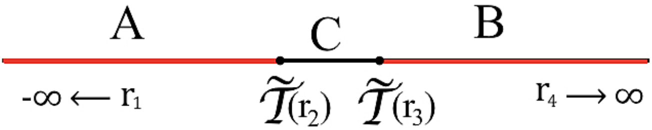

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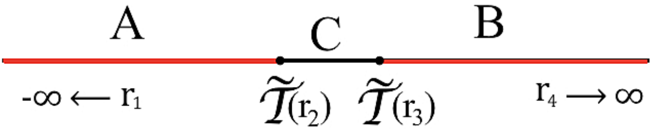

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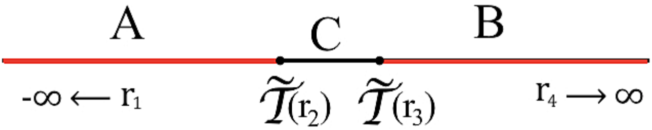

- Our aim was to investigate the leading contribution to these functions for large r. This can be accessed from the two-particle form factors of twist fields.

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- Universal, exponentially decaying corrections to this saturation are obtained by using a form factor expansion of the two-point function. Although the computation is rather technical, the key ideas involved are quite simple.

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- The first non-trivial correction to the two point function $\langle\mathcal{T}(0)\tilde{\mathcal{T}}^2(r)\rangle_n$ comes from the two particle form factor and is given by the sum

$$n\sum_{j=0}^{\frac{n}{2}-1} F(\theta+4\pi ij, n)F(\theta+2\pi ij, \frac{n}{2})$$

$$= \frac{n}{2}F(\frac{i\pi-3\theta}{2}, \frac{n}{2})\tan\left(\frac{i\theta+\pi}{4}\right) + nF(2i\pi-3\theta, n)\tan\left(\frac{i\theta}{2}\right) - (\theta\to-\theta)$$

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- We have considered the FF expansion of the following correlators

$$g(r) = \frac{\langle \mathcal{T}(0)\mathcal{T}(r)\rangle}{\langle \mathcal{T}\rangle^2} \qquad f(r) = \frac{\langle \mathcal{T}(0)\tilde{\mathcal{T}}(r)\rangle}{\langle \mathcal{T}\rangle^2}$$

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For short distances, CFT predicts that:

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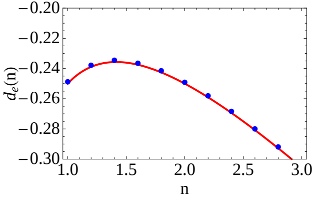

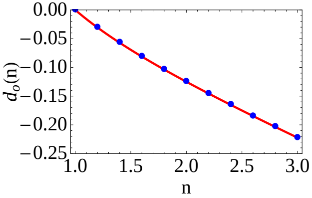

Some Numerics

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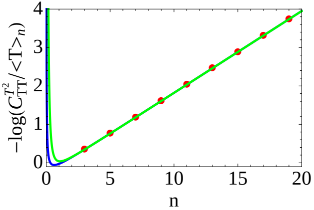
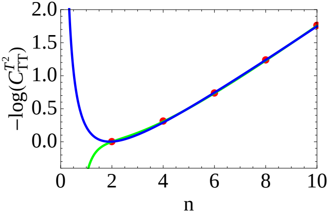
$$\log(g(r)) = \begin{cases} \underbrace{-2\Delta_n}_{d_o(n)} \log(mr) + \log\left(\frac{\mathcal{C}^{\mathcal{T}^2}_{\mathcal{T}\mathcal{T}}}{\langle\mathcal{T}\rangle_n}\right) & \text{for n odd} \\ \underbrace{-4(\Delta_n - \Delta_{\frac{n}{2}})}_{d_e(n)} \log(mr) + \log\left(\frac{\langle\mathcal{T}\rangle^2_{\frac{n}{2}} \mathcal{C}^{\mathcal{T}^2}_{\mathcal{T}\mathcal{T}}}{\langle\mathcal{T}\rangle^2_n}\right) & \text{for n even} \end{cases}$$

$$\log(f(r)) = -4\Delta_n \log(mr) - 2\log(\langle\mathcal{T}\rangle_n)$$

For the free Boson theory, FFs provide very precise numerical estimates of all these quantities





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- Much remains to be understood about the EE and LN of generic configurations: interacting theories, higher dimensions...