Entanglement Entropy of non-Unitary Quantum Field Theory

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New Trends in Strongly Entangled Many-Body Systems
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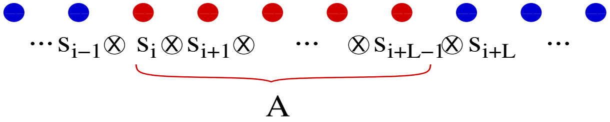

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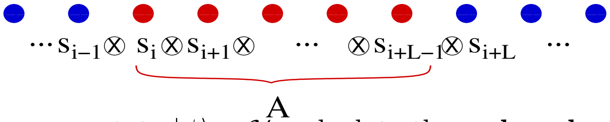

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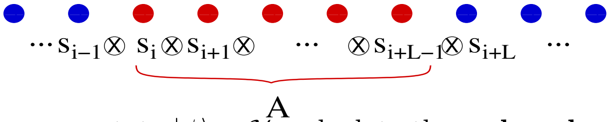

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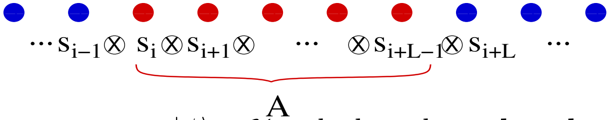

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\text{“Replica Trick”} \quad &: \quad = -\lim_{n\to 1} \frac{d}{dn} \mathrm{Tr}_{\mathcal{A}}(\rho_A^n)
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- We may tell unitary and non-unitary critical systems apart by studying the EE near criticality!

Numerical Evidence

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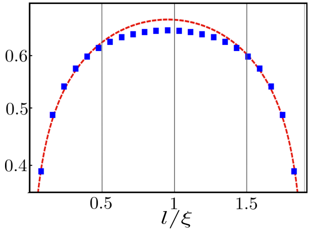
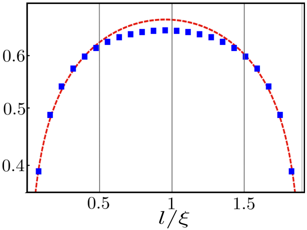

0.6
0.5
0.4
0.5
1
1.5
l/ξ



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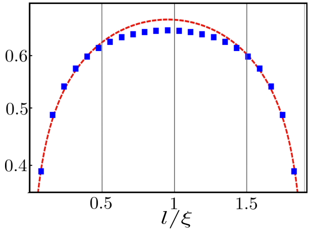

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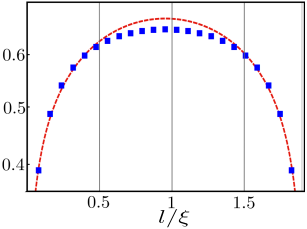

- Here ℓ is the length of the block. The data are obtained by exact diagonalization for $\lambda = 0.9$ and $N = 24$. The dashed line is the fitting curve

$$\frac{\alpha}{3} \log\left(\frac{N}{\pi} \sin \frac{\pi L}{N}\right) + \beta$$

where the finite-volume form is used [Holzhey, Larsen, Wilczek'94; Calabrese, Cardy'04]. Fitting gives

$$\alpha = 0.4056, \qquad \beta = 0.3952.$$

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- At criticality they seem to be the same: $|\psi_R\rangle = |\psi_L\rangle$. This is because they are not only `PT`-symmetric, but also `P`-symmetric.

- In radial quantization $z = e^{i\eta+\tau}$ where η is space and τ is Euclidean time we represent $|\psi_R\rangle$ by the field $\phi(0)|0\rangle = \phi_+(0)\phi_-(0)|0\rangle$ (chiral decomposition). $z \mapsto \bar{z}, \phi_+ \mapsto \phi_-$ is Parity symmetry. Hence $\mathrm{P}|\psi_R\rangle = |\psi_R\rangle$ and $\mathrm{P}|\psi_L\rangle = |\psi_L\rangle$.

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  Checked numerically and from lattice model constructions.
- We believe this may be a feature that extends to the near critical behaviour.

CFT results

We have shown that the Rényi entropy of an interval of length L starting at the boundary is given by

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- At critical points a geometric description, Riemann uniformization techniques and standard expressions for CFT partition functions is all that is needed.
- Near critical points, the scaling limit is described by massive QFT. CFT techniques fail.
- Thus if we want to go beyond criticality, a field theoretical approach to the EE becomes very powerful: twist fields

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- It has been known for some time that a "twist field" may be associated to the $\mathbb{Z}_n$ symmetry of an orbifolded CFT constructed as n cyclicly connected copies of a CFT. The conformal dimension of such field $\mathcal{T}$ was found in [Dixon, Friedan, Martinec & Shenker'87; Knizhnik'87] : $\Delta_{\mathcal{T}} = \frac{c}{24}\left(n - \frac{1}{n}\right)$.

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- In 2008 we proposed [Cardy, O.C.-A. & Doyon'08] an interpretation of the fields found in [Calabrese & Cardy'04] as branch point twist fields associated to the cyclic permutation symmetry of the replica QFT.

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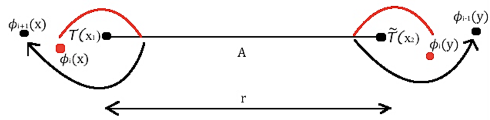
- Diagramatically:



EE and Correlation Functions in Unitary Theories

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Entanglement Entropy in Unitary QFT

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- This formulation also allows us to find sub-leading corrections to saturation (FF approach). This is what makes twist fields a powerful tool.

- Our CFT investigation has led us to conclude that for non-unitary theories the EE should rather be given by

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- ϕ is the primary field of lowest (negative) conformal dimension (e.g. the CFT ground state is created by ϕ acting on the conformal vacuum).

Generalization to Massive QFT: the Lee-Yang Model

- We now want to compute the EE for a simple massive quantum field theory. The ideal model to look at is the Lee-Yang theory with S-matrix [Cardy & Mussardo'89]

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- Form factors were computed in [Zamolodchikov'91]. He was then able to compute $\langle \phi(r)\phi(0)\rangle$ with great precision and to match results to a perturbed CFT computation.

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  where $f(\theta, n)$ is a known model-dependent function.
- Setting $n = 1$ gives either $F_1^- = 0$ or $F_1^+ = F_1^\phi$. This is strong indication that the FFs do indeed correspond to $\mathcal{O}_- = \mathcal{T}$ and $\mathcal{O}_+ =: \mathcal{T}\phi :$.

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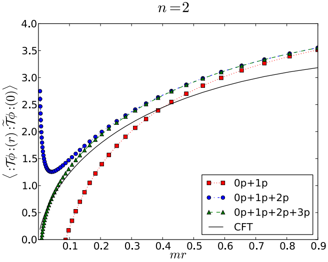
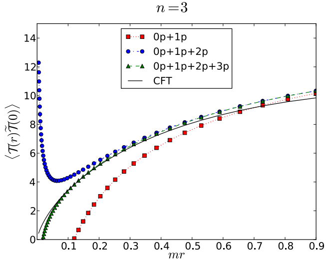
$$= r^{\frac{3}{2}}\left(1-(6.2515...)(mr)^{-\frac{2}{5}}+(8.5055...)(mr)^{-\frac{4}{5}}\right)+\cdots$$

Numerical Evidence





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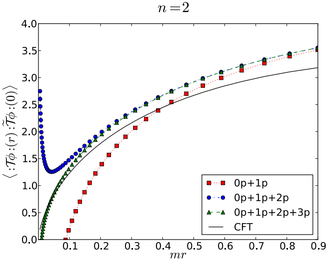

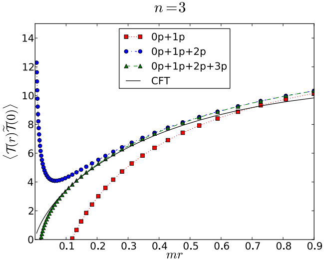

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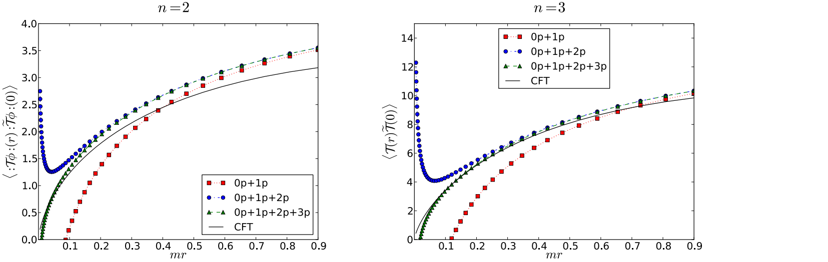

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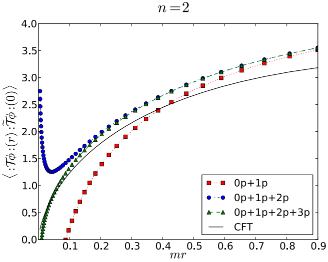

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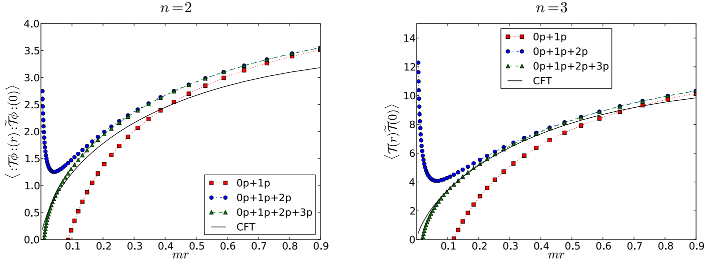

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- For very small mr the perturbed CFT results should be trusted whereas for large mr the form factor results should be the most accurate.
- The form factor results contain a further uncertainty since the values of $\langle \mathcal{T} \rangle$ and $\langle : \mathcal{T}\phi : \rangle$ are not known exactly.

Corrections to EE saturation at large regions

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- Is there an "entropic" c_{eff}-theorem? [Casini, Huerta'06]
- This provides further motivation for developing a better understanding of twist field OPEs in replica CFTs, which also plays a role in the characterisation of the negativity.