Universal Quantum Field Theory Quantities from Measures of Entanglement: The Logarithmic Negativity

Olalla A. Castro-Alvaredo

School of Mathematics, Computer Science and Engineering
Department of Mathematics
City University London

UKCFT Meeting, King's College London
11 June 2016

Related Papers:

Related Papers:

- This talk is mainly based on the papers:

  Olivier Blondeau-Fournier, O. C.-A. and Benjamin Doyon, *Universal scaling of the logarithmic negativity in massive quantum field theory*, J.Phys. A49 (2016) 125401. arXiv:1508.04026

Related Papers:

- This talk is mainly based on the papers:

  Olivier Blondeau-Fournier, O. C.-A. and Benjamin Doyon, *Universal scaling of the logarithmic negativity in massive quantum field theory*, J.Phys. A49 (2016) 125401. arXiv:1508.04026

  Davide Bianchini, Olivier Blondeau-Fournier, O. C.-A. and Benjamin Doyon, *Entanglement Measures in the 1+1 Dimensional Massive Free Boson Theory*, in preparation.

Entanglement Measures

- Consider the Bell state: $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$.

- Consider the Bell state: $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$. We all know this is a state of two particles which are maximally entangled.

Entanglement Measures

- Consider the Bell state: $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$. We all know this is a state of two particles which are maximally entangled.
- It is easy to understand and quantify the entanglement in a state as simple as the above.

Entanglement Measures

- Consider the Bell state: $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$. We all know this is a state of two particles which are maximally entangled.
- It is easy to understand and quantify the entanglement in a state as simple as the above. But what happens in extended many-body quantum systems?

Entanglement Measures

- Consider the Bell state: $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$. We all know this is a state of two particles which are maximally entangled.
- It is easy to understand and quantify the entanglement in a state as simple as the above. But what happens in extended many-body quantum systems?
- What provides a good measure of entanglement? [Plenio & Virmani'05]

Entanglement Measures

- Consider the Bell state: $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$. We all know this is a state of two particles which are maximally entangled.
- It is easy to understand and quantify the entanglement in a state as simple as the above. But what happens in extended many-body quantum systems?
- What provides a good measure of entanglement? [Plenio & Virmani'05]
    1. Should be an entanglement monotone: should not increase under LOCC

Entanglement Measures

- Consider the Bell state: $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$. We all know this is a state of two particles which are maximally entangled.
- It is easy to understand and quantify the entanglement in a state as simple as the above. But what happens in extended many-body quantum systems?
- What provides a good measure of entanglement? [Plenio & Virmani'05]
  1. Should be an entanglement monotone: should not increase under LOCC
  2. Should be invariant under unitary transformations

Entanglement Measures

- Consider the Bell state: $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$. We all know this is a state of two particles which are maximally entangled.
- It is easy to understand and quantify the entanglement in a state as simple as the above. But what happens in extended many-body quantum systems?
- What provides a good measure of entanglement? [Plenio & Virmani'05]
  1. Should be an entanglement monotone: should not increase under LOCC
  2. Should be invariant under unitary transformations
  3. Should vanish for separable states

Entanglement Measures

- Consider the Bell state: $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$. We all know this is a state of two particles which are maximally entangled.
- It is easy to understand and quantify the entanglement in a state as simple as the above. But what happens in extended many-body quantum systems?
- What provides a good measure of entanglement? [Plenio & Virmani'05]
  1. Should be an entanglement monotone: should not increase under LOCC
  2. Should be invariant under unitary transformations
  3. Should vanish for separable states
  4. Should not vanish for non-separable states

Entanglement Measures

- Consider the Bell state: $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$. We all know this is a state of two particles which are maximally entangled.
- It is easy to understand and quantify the entanglement in a state as simple as the above. But what happens in extended many-body quantum systems?
- What provides a good measure of entanglement? [Plenio & Virmani'05]
- The bi-partite entanglement entropy [Bennett et al.'96] and the logarithmic negativity [Peres'96; Eisert'00; Vidal & Werner'01; Plenio'05] are good measures of entanglement according to these properties

Logarithmic Negativity (LN)

- The LN provides a good measure of entanglement in pure and mixed states for non-complementary regions such as A and B [Peres'96; Eisert'00; Vidal & Werner'01; Plenio'05]

Logarithmic Negativity (LN)

- The LN provides a good measure of entanglement in pure and mixed states for non-complementary regions such as A and B [Peres'96; Eisert'00; Vidal & Werner'01; Plenio'05]

Logarithmic Negativity (LN)

- The LN provides a good measure of entanglement in pure and mixed states for non-complementary regions such as A and B [Peres'96; Eisert'00; Vidal & Werner'01; Plenio'05]

Logarithmic Negativity (LN)

- The LN provides a good measure of entanglement in pure and mixed states for non-complementary regions such as A and B [Peres'96; Eisert'00; Vidal & Werner'01; Plenio'05]

Logarithmic Negativity (LN)

- The LN provides a good measure of entanglement in pure and mixed states for non-complementary regions such as A and B [Peres'96; Eisert'00; Vidal & Werner'01; Plenio'05]

Logarithmic Negativity

$$\mathcal{E} = \log \mathrm{Tr}_{A\cup B}|\rho_{A\cup B}^{T_B}| \quad \text{with} \quad \rho_{A\cup B} = \mathrm{Tr}_C(|\Psi\rangle\langle\Psi|)$$

- Where $\mathrm{Tr}|\rho|$ represents the sum of the absolute values of the eigenvalues of ρ and T_B represents "partial transposition" in sub-system B

Logarithmic Negativity (LN)

- The LN provides a good measure of entanglement in pure and mixed states for non-complementary regions such as A and B [Peres'96; Eisert'00; Vidal & Werner'01; Plenio'05]

Logarithmic Negativity

$$\mathcal{E} = \log \mathrm{Tr}_{A\cup B}|\rho_{A\cup B}^{T_B}| \quad \text{with} \quad \rho_{A\cup B} = \mathrm{Tr}_C(|\Psi\rangle\langle\Psi|)$$

- Where $\mathrm{Tr}|\rho|$ represents the sum of the absolute values of the eigenvalues of ρ and T_B represents "partial transposition" in sub-system B
- $|\Psi\rangle$ is the state of the whole system (for pure states)

Logarithmic Negativity (LN)

- The LN provides a good measure of entanglement in pure and mixed states for non-complementary regions such as A and B [Peres'96; Eisert'00; Vidal & Werner'01; Plenio'05]

- There is also a "replica" approach to the computation of the negativity [Calabrese, Cardy & Tonni'12]:

Logarithmic Negativity from the Replica Trick

$$\mathcal{E}[n] = \log \mathrm{Tr}_{A\cup B}(\rho_{A\cup B}^{T_B})^n \quad \text{then} \quad \mathcal{E} = \lim_{n\to 1} \mathcal{E}_e[n]$$

where $\mathcal{E}_e[n]$ means the function $\mathcal{E}[n]$ for n even. This limit requires analytic continuation from n even to $n = 1$

What QFT information is contained in the LN?

At criticality:

What QFT information is contained in the LN?

At criticality:

Universal scaling: For "adjacent regions" [Calabrese, Cardy & Tonni'12'13'14]:

$$\mathcal{E}^{\perp}(\ell_1, \ell_2) \sim \frac{c}{4} \log \frac{\ell_1 \ell_2}{\ell_1 + \ell_2}$$

c is the central change.

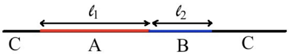
What QFT information is contained in the LN?

At criticality:

Universal scaling: For "adjacent regions" [Calabrese, Cardy & Tonni'12'13'14]:

$$\mathcal{E}^{\perp}(\ell_1, \ell_2) \sim \frac{c}{4} \log \frac{\ell_1 \ell_2}{\ell_1 + \ell_2}$$

c is the central change.



For general confs: information about operator content of CFT.

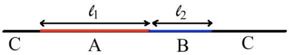
What QFT information is contained in the LN?

At criticality:

Universal scaling: For "adjacent regions" [Calabrese, Cardy & Tonni'12'13'14]:

$$\mathcal{E}^{\perp}(\ell_1, \ell_2) \sim \frac{c}{4} \log \frac{\ell_1 \ell_2}{\ell_1 + \ell_2}$$

c is the central change.



For general confs: information about operator content of CFT. Best known for compactified free Boson.

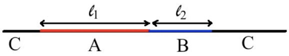
What QFT information is contained in the LN?

At criticality:

Universal scaling: For "adjacent regions" [Calabrese, Cardy & Tonni'12'13'14]:

$$\mathcal{E}^{\perp}(\ell_1, \ell_2) \sim \frac{c}{4} \log \frac{\ell_1 \ell_2}{\ell_1 + \ell_2}$$

c is the central change.



For general confs: information about operator content of CFT. Best known for compactified free Boson.

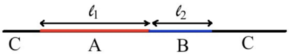
What QFT information is contained in the LN?

At criticality:

Universal scaling: For "adjacent regions" [Calabrese, Cardy & Tonni'12'13'14]:

$$\mathcal{E}^{\perp}(\ell_1, \ell_2) \sim \frac{c}{4} \log \frac{\ell_1 \ell_2}{\ell_1 + \ell_2}$$

c is the central change.



For general confs: information about operator content of CFT. Best known for compactified free Boson.

Near criticality:

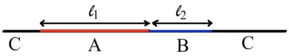
What QFT information is contained in the LN?

At criticality:

Universal scaling: For "adjacent regions" [Calabrese, Cardy & Tonni'12'13'14]:

$$\mathcal{E}^{\perp}(\ell_1, \ell_2) \sim \frac{c}{4} \log \frac{\ell_1 \ell_2}{\ell_1 + \ell_2}$$

c is the central change.



For general confs: information about operator content of CFT. Best known for compactified free Boson.

Near criticality:

Universal saturation and decay: For adjacent regions ($\ell_1 := \ell$, $\ell_2 \to \infty$) [Blondeau-Fournier, OC-A & Doyon'15]

$$\mathcal{E}^{\perp}(\ell) \sim -\frac{c}{4}\log(m\epsilon) + \mathcal{E}_{\text{sat}} - \frac{2a}{3\sqrt{3}\pi} K_0(\sqrt{3}m\ell)$$

where $m \propto \xi^{-1}$ is the smallest mass scale in the theory

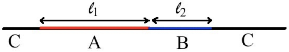
What QFT information is contained in the LN?

At criticality:

Universal scaling: For "adjacent regions" [Calabrese, Cardy & Tonni'12'13'14]:

$$\mathcal{E}^{\perp}(\ell_1,\ell_2) \sim \frac{c}{4}\log\frac{\ell_1\ell_2}{\ell_1+\ell_2}$$

c is the central change.



For general confs: information about operator content of CFT. Best known for compactified free Boson.

Near criticality:

Universal saturation and decay: For adjacent regions ($\ell_1 := \ell$, $\ell_2 \to \infty$) [Blondeau-Fournier, OC-A & Doyon'15]

$$\mathcal{E}^{\perp}(\ell) \sim -\frac{c}{4}\log(m\epsilon) + \mathcal{E}_{\rm sat} - \frac{2a}{3\sqrt{3}\pi}K_0(\sqrt{3}m\ell)$$

where $m \propto \xi^{-1}$ is the smallest mass scale in the theory a is the number of lightest particles,

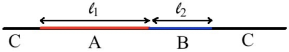
What QFT information is contained in the LN?

At criticality:

Universal scaling: For "adjacent regions" [Calabrese, Cardy & Tonni'12'13'14]:

$$\mathcal{E}^{\perp}(\ell_1, \ell_2) \sim \frac{c}{4} \log \frac{\ell_1 \ell_2}{\ell_1 + \ell_2}$$

c is the central change.



For general confs: information about operator content of CFT. Best known for compactified free Boson.

Near criticality:

Universal saturation and decay: For adjacent regions ($\ell_1 := \ell$, $\ell_2 \to \infty$) [Blondeau-Fournier, OC-A & Doyon'15]

$$\mathcal{E}^{\perp}(\ell) \sim -\frac{c}{4} \log(m\epsilon) + \mathcal{E}_{\text{sat}} - \frac{2a}{3\sqrt{3}\pi} K_0(\sqrt{3} m \ell)$$

where $m \propto \xi^{-1}$ is the smallest mass scale in the theory a is the number of lightest particles, ϵ is a short distance cut-off

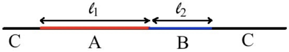
What QFT information is contained in the LN?

At criticality:

Universal scaling: For "adjacent regions" [Calabrese, Cardy & Tonni'12'13'14]:

$$\mathcal{E}^{\perp}(\ell_1,\ell_2) \sim \frac{c}{4}\log\frac{\ell_1\ell_2}{\ell_1+\ell_2}$$

c is the central change.



For general confs: information about operator content of CFT. Best known for compactified free Boson.

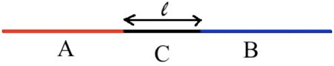
Near criticality:

Universal saturation and decay: For adjacent regions ($\ell_1 := \ell$, $\ell_2 \to \infty$) [Blondeau-Fournier, OC-A & Doyon'15]

$$\mathcal{E}^{\perp}(\ell) \sim -\frac{c}{4}\log(m\epsilon)+\mathcal{E}_{\rm sat}-\frac{2a}{3\sqrt{3}\pi}K_0(\sqrt{3}m\ell)$$

where $m \propto \xi^{-1}$ is the smallest mass scale in the theory a is the number of lightest particles, ϵ is a short distance cut-off and $\mathcal{E}_{\rm sat}$ is a universal constant.

For semi-infinite non-adjacent regions:



$$\mathcal{E}^{\dashv\vdash}(\ell) \sim \frac{a(m\ell)^2}{2\pi^2}\left[K_0(m\ell)^2+\frac{K_0(m\ell)K_1(m\ell)}{m\ell}-K_1(m\ell)^2\right]$$

Twist Fields & Entanglement Measures

- A description of the EE of a single interval as a two-point function of a special field with a particular conformal dimension first appeared in [Calabrese & Cardy'04]

Twist Fields & Entanglement Measures

- A description of the EE of a single interval as a two-point function of a special field with a particular conformal dimension first appeared in [Calabrese & Cardy'04]
- This kind of field had previously appeared in the study of orbifold CFT and its conformal dimension was known [Knizhnik'87; Dixon et al.'87]

$$\Delta_n = \frac{c}{24}\left(n - \frac{1}{n}\right)$$

Twist Fields & Entanglement Measures

- A description of the EE of a single interval as a two-point function of a special field with a particular conformal dimension first appeared in [Calabrese & Cardy'04]
- This kind of field had previously appeared in the study of orbifold CFT and its conformal dimension was known [Knizhnik'87; Dixon et al.'87]

$$\Delta_n = \frac{c}{24}\left(n - \frac{1}{n}\right)$$

- We later proposed an interpretation of this field as a branch point twist field and characterize it through its locality properties versus other fields in the replica theory [Cardy, O.C-A & Doyon'08]

QFT Definition of Twist Fields

- The Twist Fields are defined through very general conmutation relations with the fundamental field of the model [Cardy, O.C-A & Doyon'08]:

$$\begin{aligned}
\Phi_i(y)\mathcal{T}(x) &= \mathcal{T}(x)\Phi_{i+1}(y) \qquad x^1 > y^1, \\
\Phi_i(y)\mathcal{T}(x) &= \mathcal{T}(x)\Phi_{i}(y) \qquad x^1 < y^1,
\end{aligned}$$

$$\begin{aligned}
\Phi_i(y)\tilde{\mathcal{T}}(x) &= \tilde{\mathcal{T}}(x)\Phi_{i-1}(y) \qquad x^1 > y^1, \\
\Phi_i(y)\tilde{\mathcal{T}}(x) &= \tilde{\mathcal{T}}(x)\Phi_{i}(y) \qquad x^1 < y^1.
\end{aligned}$$

for $i = 1, \ldots, n$ and $n + i \equiv i$.

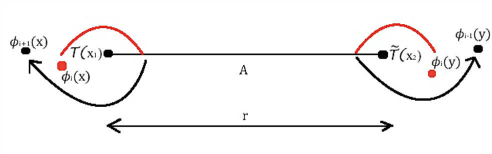
QFT Definition of Twist Fields

- The Twist Fields are defined through very general conmutation relations with the fundamental field of the model [Cardy, O.C-A & Doyon'08]:

$$
\begin{aligned}
\Phi_i(y)\mathcal{T}(x) &= \mathcal{T}(x)\Phi_{i+1}(y) \qquad x^1 > y^1, \\
\Phi_i(y)\mathcal{T}(x) &= \mathcal{T}(x)\Phi_i(y) \qquad x^1 < y^1, \\
\\
\Phi_i(y)\tilde{\mathcal{T}}(x) &= \tilde{\mathcal{T}}(x)\Phi_{i-1}(y) \qquad x^1 > y^1, \\
\Phi_i(y)\tilde{\mathcal{T}}(x) &= \tilde{\mathcal{T}}(x)\Phi_i(y) \qquad x^1 < y^1.
\end{aligned}
$$

  for $i = 1, \ldots, n$ and $n + i \equiv i$.

- Diagramatically:



Logarithmic Negativity from Twist Fields

- The twist field approach has been used in the study of the LN of CFT [Calabrese, Cardy & Tonni'12'13'14]

Logarithmic Negativity from Twist Fields

- The twist field approach has been used in the study of the LN of CFT [Calabrese, Cardy & Tonni'12'13'14]

Logarithmic Negativity from Twist Fields

$$\mathcal{E}[n] = \log\left(\varepsilon^{8\Delta_n}\langle\mathcal{T}(r_1)\tilde{\mathcal{T}}(r_2)\tilde{\mathcal{T}}(r_3)\mathcal{T}(r_4)\rangle_n\right)$$

Logarithmic Negativity from Twist Fields

- The twist field approach has been used in the study of the LN of CFT [Calabrese, Cardy & Tonni'12'13'14]

Logarithmic Negativity from Twist Fields

$$\mathcal{E}[n] = \log \left(\varepsilon^{8\Delta_n} \langle \mathcal{T}(r_1)\tilde{\mathcal{T}}(r_2)\tilde{\mathcal{T}}(r_3)\mathcal{T}(r_4)\rangle_n \right)$$

where $|r_2 - r_1|$ = length of A and $|r_4 - r_3|$ = length of B

Logarithmic Negativity from Twist Fields

- The twist field approach has been used in the study of the LN of CFT [Calabrese, Cardy & Tonni'12'13'14]

Logarithmic Negativity from Twist Fields

$$\mathcal{E}[n] = \log\left(\varepsilon^{8\Delta_n}\langle \mathcal{T}(r_1)\tilde{\mathcal{T}}(r_2)\tilde{\mathcal{T}}(r_3)\mathcal{T}(r_4)\rangle_n\right)$$

where $|r_2 - r_1|$ = length of A and $|r_4 - r_3|$ = length of B

- This 4-point function has been investigated in CFT but the analytic continuation remains challenging, even for free theories

Logarithmic Negativity from Twist Fields

- The twist field approach has been used in the study of the LN of CFT [Calabrese, Cardy & Tonni'12'13'14]

Logarithmic Negativity from Twist Fields

$$\mathcal{E}[n] = \log\left(\varepsilon^{8\Delta_n}\langle\mathcal{T}(r_1)\tilde{\mathcal{T}}(r_2)\tilde{\mathcal{T}}(r_3)\mathcal{T}(r_4)\rangle_n\right)$$

where $|r_2 - r_1|$ = length of A and $|r_4 - r_3|$ = length of B

- This 4-point function has been investigated in CFT but the analytic continuation remains challenging, even for free theories
- An interesting limit is $\lim_{r_2\to r_3}\tilde{\mathcal{T}}(r_2)\tilde{\mathcal{T}}(r_3) \sim \tilde{\mathcal{T}}^2(r_3)$ where $\tilde{\mathcal{T}}^2$ is defined as the twist field associated to the cyclic permutation $j \mapsto j-2$.

Logarithmic Negativity from Twist Fields

- The twist field approach has been used in the study of the LN of CFT [Calabrese, Cardy & Tonni'12'13'14]

Logarithmic Negativity from Twist Fields

$$\mathcal{E}[n] = \log\left(\varepsilon^{8\Delta_n}\langle \mathcal{T}(r_1)\tilde{\mathcal{T}}(r_2)\tilde{\mathcal{T}}(r_3)\mathcal{T}(r_4)\rangle_n\right)$$

where $|r_2 - r_1|$ = length of A and $|r_4 - r_3|$ = length of B

- This 4-point function has been investigated in CFT but the analytic continuation remains challenging, even for free theories
- An interesting limit is $\lim_{r_2\to r_3} \tilde{\mathcal{T}}(r_2)\tilde{\mathcal{T}}(r_3) \sim \tilde{\mathcal{T}}^2(r_3)$ where $\tilde{\mathcal{T}}^2$ is defined as the twist field associated to the cyclic permutation $j \mapsto j-2$. $\mathcal{T}_n^2 = \mathcal{T}_{\frac{n}{2}} \otimes \mathcal{T}_{\frac{n}{2}}$ (n even)

Logarithmic Negativity from Twist Fields

- The twist field approach has been used in the study of the LN of CFT [Calabrese, Cardy & Tonni'12'13'14]

Logarithmic Negativity from Twist Fields

$$\mathcal{E}[n] = \log\left(\varepsilon^{8\Delta_n}\langle\mathcal{T}(r_1)\tilde{\mathcal{T}}(r_2)\tilde{\mathcal{T}}(r_3)\mathcal{T}(r_4)\rangle_n\right)$$

where $|r_2 - r_1|$ = length of A and $|r_4 - r_3|$ = length of B

- This 4-point function has been investigated in CFT but the analytic continuation remains challenging, even for free theories
- An interesting limit is $\lim_{r_2\to r_3}\tilde{\mathcal{T}}(r_2)\tilde{\mathcal{T}}(r_3) \sim \tilde{\mathcal{T}}^2(r_3)$ where $\tilde{\mathcal{T}}^2$ is defined as the twist field associated to the cyclic permutation $j \mapsto j-2$. $\mathcal{T}_n^2 = \mathcal{T}_{\frac{n}{2}} \otimes \mathcal{T}_{\frac{n}{2}}$ (n even) $\mathcal{T}^2 = \mathcal{T}$ (n odd)

LN in Massive QFT: Adjacent Regions

- In our work we have studied two simple limits of the LN in a completely generic 1+1 dimensional QFT

LN in Massive QFT: Adjacent Regions

- In our work we have studied two simple limits of the LN in a completely generic 1+1 dimensional QFT
- *Adjacent regions* (one semi-infinite region): $r_3 \to r_2 := r$ and $r_4 \to \infty$ and we will choose $r_1 = 0$

$$\mathcal{E}_e^{\perp}[n] = \log\left(\varepsilon^{4\Delta_n + 4\Delta_{\frac{n}{2}}} \langle \mathcal{T}(0)\tilde{\mathcal{T}}^2(r)\rangle_n \langle \mathcal{T}\rangle_n\right)$$

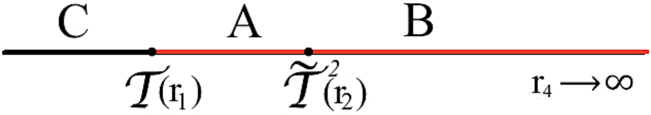
$2\Delta_{\frac{n}{2}}$ is the conformal dimension of $\mathcal{T}^2$ for n even.

LN in Massive QFT: Adjacent Regions

- In our work we have studied two simple limits of the LN in a completely generic 1+1 dimensional QFT
- *Adjacent regions* (one semi-infinite region): $r_3 \to r_2 := r$ and $r_4 \to \infty$ and we will choose $r_1 = 0$

$$\mathcal{E}_e^{\perp}[n] = \log\left(\varepsilon^{4\Delta_n + 4\Delta_{\frac{n}{2}}} \langle \mathcal{T}(0)\tilde{\mathcal{T}}^2(r)\rangle_n \langle \mathcal{T}\rangle_n\right)$$

$2\Delta_{\frac{n}{2}}$ is the conformal dimension of $\mathcal{T}^2$ for n even.



LN in Massive QFT: Disjoint Regions

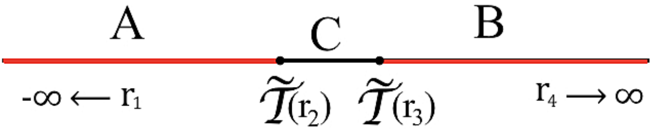
- *Disjoint semi-infinite regions* : $r_1 \to -\infty$, $r_4 \to \infty$, and we will choose $r_2 = 0$, $r_3 = r$

$$\mathcal{E}_e^{\dashv\vdash}[n] = \log\left(\varepsilon^{8\Delta_n}\langle\mathcal{T}\rangle_n\langle\tilde{\mathcal{T}}(0)\tilde{\mathcal{T}}(r)\rangle_n\langle\mathcal{T}\rangle_n\right)$$

LN in Massive QFT: Disjoint Regions

- *Disjoint semi-infinite regions* : $r_1 \to -\infty$, $r_4 \to \infty$, and we will choose $r_2 = 0$, $r_3 = r$

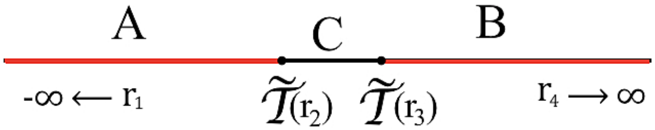
$$\mathcal{E}_e^{\dashv\vdash}[n] = \log\left(\varepsilon^{8\Delta_n}\langle\mathcal{T}\rangle_n\langle\tilde{\mathcal{T}}(0)\tilde{\mathcal{T}}(r)\rangle_n\langle\mathcal{T}\rangle_n\right)$$



LN in Massive QFT: Disjoint Regions

- *Disjoint semi-infinite regions* : $r_1 \to -\infty$, $r_4 \to \infty$, and we will choose $r_2 = 0$, $r_3 = r$

$$\mathcal{E}_e^{\dashv\vdash}[n] = \log\left(\varepsilon^{8\Delta_n}\langle\mathcal{T}\rangle_n\langle\tilde{\mathcal{T}}(0)\tilde{\mathcal{T}}(r)\rangle_n\langle\mathcal{T}\rangle_n\right)$$

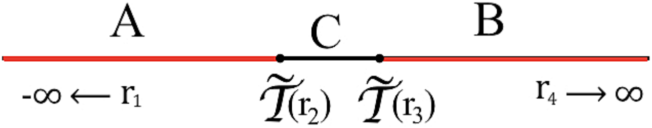


- Our aim was to investigate the leading contribution to these functions for large r.

LN in Massive QFT: Disjoint Regions

- *Disjoint semi-infinite regions* : $r_1 \to -\infty$, $r_4 \to \infty$, and we will choose $r_2 = 0$, $r_3 = r$

$$\mathcal{E}_e^{\dashv\vdash}[n] = \log\left(\varepsilon^{8\Delta_n}\langle\mathcal{T}\rangle_n\langle\tilde{\mathcal{T}}(0)\tilde{\mathcal{T}}(r)\rangle_n\langle\mathcal{T}\rangle_n\right)$$



- Our aim was to investigate the leading contribution to these functions for large r. This can be accessed from the two-particle form factors of twist fields.

Universal QFT Quantities from LN

- As we have seen, various limits of the LN are related to two-point functions of twist fields.

- As we have seen, various limits of the LN are related to two-point functions of twist fields. These in turn contain information about the VEV and three-point couplings of branch point twist fields.

Universal QFT Quantities from LN

- As we have seen, various limits of the LN are related to two-point functions of twist fields. These in turn contain information about the VEV and three-point couplings of branch point twist fields.
- For the free Boson we can obtain accurate predictions for these quantities and/or their ratios by employing a FF expansion (all twist field FFs are known in this case) [Bianchini, Blondeau-Fournier, OC-A and Doyon'16]

Universal QFT Quantities from LN

- As we have seen, various limits of the LN are related to two-point functions of twist fields. These in turn contain information about the VEV and three-point couplings of branch point twist fields.
- For the free Boson we can obtain accurate predictions for these quantities and/or their ratios by employing a FF expansion (all twist field FFs are known in this case) [Bianchini, Blondeau-Fournier, OC-A and Doyon'16]
- We have considered the FF expansion of the following correlators

$$g(r) = \frac{\langle \mathcal{T}(0)\mathcal{T}(r)\rangle}{\langle \mathcal{T}\rangle^2} \qquad f(r) = \frac{\langle \mathcal{T}(0)\tilde{\mathcal{T}}(r)\rangle}{\langle \mathcal{T}\rangle^2}$$

Some Numerics

For short distances, CFT predicts that:

$$\log(g(r)) = \begin{cases} \underbrace{-2\Delta_n}_{d_o(n)} \log(r) + \log\left(\frac{\mathcal{C}_{\mathcal{T}\tilde{\mathcal{T}}}^{\mathcal{T}^2}}{\langle\mathcal{T}\rangle_n}\right) & \text{for n odd} \\ \underbrace{-4(\Delta_n - \Delta_{\frac{n}{2}})}_{d_e(n)} \log(r) + \log\left(\frac{\langle\mathcal{T}\rangle_{\frac{n}{2}}^2 \mathcal{C}_{\mathcal{T}\tilde{\mathcal{T}}}^{\mathcal{T}^2}}{\langle\mathcal{T}\rangle_n^2}\right) & \text{for n even} \end{cases}$$

Some Numerics

For short distances, CFT predicts that:

$$\log(g(r)) = \begin{cases} \underbrace{-2\Delta_n}_{d_o(n)} \log(r) + \log\left(\frac{\mathcal{C}^{\mathcal{T}^2}_{\mathcal{T}\mathcal{T}}}{\langle \mathcal{T} \rangle_n}\right) & \text{for n odd} \\ \underbrace{-4(\Delta_n - \Delta_{\frac{n}{2}})}_{d_e(n)} \log(r) + \log\left(\frac{\langle \mathcal{T} \rangle^2_{\frac{n}{2}} \mathcal{C}^{\mathcal{T}^2}_{\mathcal{T}\mathcal{T}}}{\langle \mathcal{T} \rangle^2_n}\right) & \text{for n even} \end{cases}$$

$$\log(f(r)) = -4\Delta_n \log(r) - 2\log(\langle \mathcal{T} \rangle_n)$$

Some Numerics

For short distances, CFT predicts that:

$$\log(g(r)) = \begin{cases} \underbrace{-2\Delta_n}_{d_o(n)} \log(r) + \log\left(\frac{\mathcal{C}_{\mathcal{T}\mathcal{T}}^{\mathcal{T}^2}}{\langle\mathcal{T}\rangle_n}\right) & \text{for n odd} \\ \underbrace{-4(\Delta_n - \Delta_{\frac{n}{2}})}_{d_e(n)} \log(r) + \log\left(\frac{\langle\mathcal{T}\rangle_{\frac{n}{2}}^2 \mathcal{C}_{\mathcal{T}\mathcal{T}}^{\mathcal{T}^2}}{\langle\mathcal{T}\rangle_n^2}\right) & \text{for n even} \end{cases}$$

$$\log(f(r)) = -4\Delta_n \log(r) - 2\log(\langle\mathcal{T}\rangle_n)$$

These expressions simply follow from

$$\langle\mathcal{T}(0)\mathcal{T}(r)\rangle \sim r^{-4\Delta_n + 2\Delta_{\mathcal{T}^2}} C_{\mathcal{T}\mathcal{T}}^{\mathcal{T}^2}$$

Some Numerics

For short distances, CFT predicts that:

$$\log(g(r)) = \begin{cases} \underbrace{-2\Delta_n}_{d_o(n)} \log(r) + \log\left(\frac{\mathcal{C}_{\mathcal{T}\mathcal{T}}^{\mathcal{T}^2}}{\langle\mathcal{T}\rangle_n}\right) & \text{for n odd} \\ \underbrace{-4(\Delta_n - \Delta_{\frac{n}{2}})}_{d_e(n)} \log(r) + \log\left(\frac{\langle\mathcal{T}\rangle_{\frac{n}{2}}^2 \mathcal{C}_{\mathcal{T}\mathcal{T}}^{\mathcal{T}^2}}{\langle\mathcal{T}\rangle_n^2}\right) & \text{for n even} \end{cases}$$

$$\log(f(r)) = -4\Delta_n \log(r) - 2\log(\langle\mathcal{T}\rangle_n)$$

These expressions simply follow from

$$\langle\mathcal{T}(0)\mathcal{T}(r)\rangle \sim r^{-4\Delta_n + 2\Delta_{\mathcal{T}^2}} C_{\mathcal{T}\mathcal{T}}^{\mathcal{T}^2}$$

and

$$\langle\mathcal{T}(0)\tilde{\mathcal{T}}(r)\rangle \sim r^{-4\Delta_n}$$

Massive Free Boson

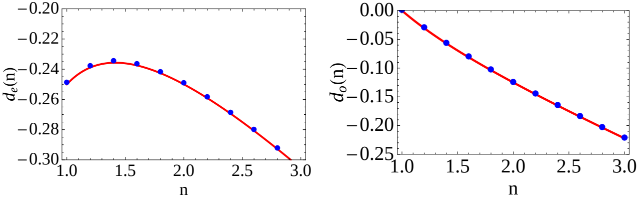
- For the free Boson theory, FFs provide very precise numerical estimates of all these quantities.

Massive Free Boson

- For the free Boson theory, FFs provide very precise numerical estimates of all these quantities. The coefficients of the $\log r$ terms can be obtained with great precision.

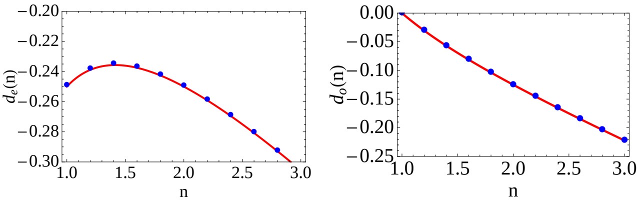
Massive Free Boson

- For the free Boson theory, FFs provide very precise numerical estimates of all these quantities. The coefficients of the $\log r$ terms can be obtained with great precision. For the correlator $g(r)$ they can in fact be evaluated exactly by summing up the full form factor expansion!



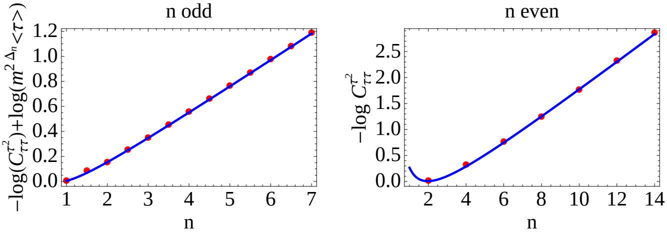
Massive Free Boson

- For the free Boson theory, FFs provide very precise numerical estimates of all these quantities. The coefficients of the $\log r$ terms can be obtained with great precision. For the correlator $g(r)$ they can in fact be evaluated exactly by summing up the full form factor expansion!

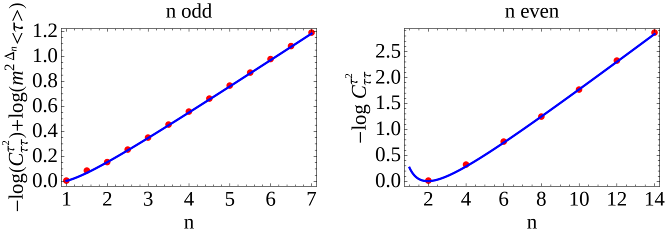


- This result does not provide new information but it provides strong support for the analytic continuation in n.
- We can also numerically evaluate the next-to-leading order correction to the two-point functions...

Expectation values and three-point couplings

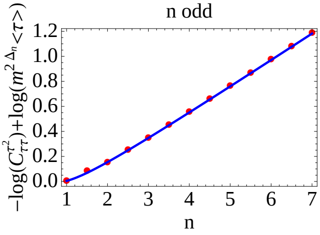


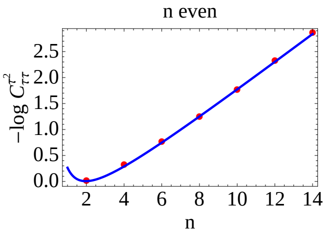
Expectation values and three-point couplings



- For n even, we can use an interpolating function of the form $a_0 + a_1 b + a_2/n$ to find the (numerical) analytic continuation of the three-point coupling at $n = 1$ (important in the LN context).

Expectation values and three-point couplings





- For n even, we can use an interpolating function of the form $a_0 + a_1 b + a_2/n$ to find the (numerical) analytic continuation of the three-point coupling at $n = 1$ (important in the LN context).
- The value we obtain is 0.7(6) which is in relatively close agreement with the value 0.832... which was computed analytically in [Calabrese, Cardy & Tonni'13']

Some detail on the numerics

- The form factors provide infinite sum representations of the form

$$\log(\langle \mathcal{T} \rangle_n) = n \sum_{j=1}^{\infty} \frac{u_j(n)}{j(4\pi)^{2j}}$$

Some detail on the numerics

- The form factors provide infinite sum representations of the form

$$\log(\langle \mathcal{T} \rangle_n) = n \sum_{j=1}^{\infty} \frac{u_j(n)}{j(4\pi)^{2j}}$$

where $u_j(n)$ are given by multiple integrals of complicated (but explicit) functions of the rapidities.

Some detail on the numerics

- The form factors provide infinite sum representations of the form

$$\log(\langle\mathcal{T}\rangle_n) = n\sum_{j=1}^{\infty}\frac{u_j(n)}{j(4\pi)^{2j}}$$

  where $u_j(n)$ are given by multiple integrals of complicated (but explicit) functions of the rapidities.
- Similarly

$$\log\left(\frac{\langle\mathcal{T}\rangle^2_{\frac{n}{2}}\mathcal{C}^{\mathcal{T}^2}_{\mathcal{T}\mathcal{T}}}{\langle\mathcal{T}\rangle^2_n}\right)_{\text{n even}} = n\sum_{j=1}^{\infty}\frac{h_e(n,j)v_j}{j(2\pi)^{2j}}$$

Some detail on the numerics

- The form factors provide infinite sum representations of the form

$$\log(\langle \mathcal{T} \rangle_n) = n \sum_{j=1}^{\infty} \frac{u_j(n)}{j(4\pi)^{2j}}$$

where $u_j(n)$ are given by multiple integrals of complicated (but explicit) functions of the rapidities.

- Similarly

$$\log\left(\frac{\langle \mathcal{T} \rangle_{\frac{n}{2}}^2 \mathcal{C}_{\mathcal{T}\mathcal{T}}^{\mathcal{T}^2}}{\langle \mathcal{T} \rangle_n^2}\right)_{\text{n even}} = n \sum_{j=1}^{\infty} \frac{h_e(n,j) v_j}{j(2\pi)^{2j}}$$

and

$$\log\left(\frac{\mathcal{C}_{\mathcal{T}\mathcal{T}}^{\mathcal{T}^2}}{\langle \mathcal{T} \rangle_n}\right)_{\text{n odd}} = n \sum_{j=1}^{\infty} \frac{h_o(n,j) v_j}{j(2\pi)^{2j}}$$

Some detail on the numerics

- The form factors provide infinite sum representations of the form

$$\log(\langle\mathcal{T}\rangle_n) = n\sum_{j=1}^{\infty}\frac{u_j(n)}{j(4\pi)^{2j}}$$

where $u_j(n)$ are given by multiple integrals of complicated (but explicit) functions of the rapidities.

- Similarly

$$\log\left(\frac{\langle\mathcal{T}\rangle_{\frac{n}{2}}^2\,\mathcal{C}_{\mathcal{T}\mathcal{T}}^{\mathcal{T}^2}}{\langle\mathcal{T}\rangle_n^2}\right)_{\text{n even}} = n\sum_{j=1}^{\infty}\frac{h_e(n,j)v_j}{j(2\pi)^{2j}}$$

and

$$\log\left(\frac{\mathcal{C}_{\mathcal{T}\mathcal{T}}^{\mathcal{T}^2}}{\langle\mathcal{T}\rangle_n}\right)_{\text{n odd}} = n\sum_{j=1}^{\infty}\frac{h_o(n,j)v_j}{j(2\pi)^{2j}}$$

where the functions $h_{e,o}(n,j)$ are known and v_j are again given as multiple integrals of complicated functions.

Some Questions

Some Questions

- An interesting aspect of the numerics is that some of the sums above are divergent!

Some Questions

- An interesting aspect of the numerics is that some of the sums above are divergent!
- However, by combining various sums the divergencies can be cancelled out and finite values for $\log C_{\mathcal{T}\tilde{\mathcal{T}}}^{\mathcal{T}^2}$ obtained.

Some Questions

- An interesting aspect of the numerics is that some of the sums above are divergent!
- However, by combining various sums the divergencies can be cancelled out and finite values for $\log C_{\mathcal{T}\mathcal{T}}^{\mathcal{T}^2}$ obtained.
- This suggests divergent vacuum expectation values and three point couplings (for n odd) but a finite ratio thereof.

Some Questions

- An interesting aspect of the numerics is that some of the sums above are divergent!
- However, by combining various sums the divergencies can be cancelled out and finite values for $\log C_{\mathcal{T}\mathcal{T}}^{\mathcal{T}^2}$ obtained.
- This suggests divergent vacuum expectation values and three point couplings (for n odd) but a finite ratio thereof.
- We think this points towards the presence of logartihmic divergencies in the correlators.

Some Questions

- An interesting aspect of the numerics is that some of the sums above are divergent!
- However, by combining various sums the divergencies can be cancelled out and finite values for $\log C_{\mathcal{T}\mathcal{T}}^{\mathcal{T}^2}$ obtained.
- This suggests divergent vacuum expectation values and three point couplings (for n odd) but a finite ratio thereof.
- We think this points towards the presence of logartihmic divergencies in the correlators. This has actually been shown to be the case for the four-point functions of twist fields.

Some Questions

- An interesting aspect of the numerics is that some of the sums above are divergent!
- However, by combining various sums the divergencies can be cancelled out and finite values for $\log C_{\mathcal{T}\mathcal{T}}^{\mathcal{T}^2}$ obtained.
- This suggests divergent vacuum expectation values and three point couplings (for n odd) but a finite ratio thereof.
- We think this points towards the presence of logartihmic divergencies in the correlators. This has actually been shown to be the case for the four-point functions of twist fields. This is a peculiarity of the free Boson theory.

Conclusion & Outlook

- Analytical computations of the LN for 1+1 dimensional QFTs remain challenging

Conclusion & Outlook

- Analytical computations of the LN for 1+1 dimensional QFTs remain challenging
- We have shown that at least in particular limits the LN exhibits remarkable universality even beyond criticality

Conclusion & Outlook

- Analytical computations of the LN for 1+1 dimensional QFTs remain challenging
- We have shown that at least in particular limits the LN exhibits remarkable universality even beyond criticality
- Large region corrections to the LN in massive theories provide information about the mass spectrum of QFT

Conclusion & Outlook

- Analytical computations of the LN for 1+1 dimensional QFTs remain challenging
- We have shown that at least in particular limits the LN exhibits remarkable universality even beyond criticality
- Large region corrections to the LN in massive theories provide information about the mass spectrum of QFT
- They also provide a means to access universal quantities in QFT such as VEVs and structure constants in CFT, at least numerically

Conclusion & Outlook

- Analytical computations of the LN for 1+1 dimensional QFTs remain challenging
- We have shown that at least in particular limits the LN exhibits remarkable universality even beyond criticality
- Large region corrections to the LN in massive theories provide information about the mass spectrum of QFT
- They also provide a means to access universal quantities in QFT such as VEVs and structure constants in CFT, at least numerically
- For the free Boson theory such universal quantities may be determined with high numerical precision, providing in some cases their first known estimates by any method

Conclusion & Outlook

- Analytical computations of the LN for 1+1 dimensional QFTs remain challenging
- We have shown that at least in particular limits the LN exhibits remarkable universality even beyond criticality
- Large region corrections to the LN in massive theories provide information about the mass spectrum of QFT
- They also provide a means to access universal quantities in QFT such as VEVs and structure constants in CFT, at least numerically
- For the free Boson theory such universal quantities may be determined with high numerical precision, providing in some cases their first known estimates by any method
- Much remains to be understood about the EE and LN of generic configurations: interacting theories, higher dimensions...