Did you know he has a Lamborghini? Francesco's Work, Life and (some) Secrets plus New Results on Minimal Form Factors

Olalla Castro-Alvaredo
Department of Mathematics
City, University of London

10th Bologna Workshop on CFT and Integrable Models, 3-8 September 2023


So….
First
Things
First

L’Apoteosi Di Francesco de Novara

I have known Francesco for a very long time.....

Our most successful collaboration …

Davide Bianchini
PhD 2013-16

Cecilia De Fazio
MSc 2017-18
PhD 2018-21

Michele Mazzoni
MSc 2020-21
PhD 2021-24?

Fabio Sailis
PhD 2022-25?

Riccardo Travaglino
MSc 2022-23

We have done some work together!

- M. Mazzoni, O. Pomponio, O.A. Castro-Alvaredo and **F. Ravanini**, The Staircase Model: Massless Flows and Hydrodynamics, J. Phys. A54 404005 (2021), arXiv:2105.13349. Part of the Special Issue on Hydrodynamics of Low-Dimensional Quantum Systems.
- O.A. Castro-Alvaredo, C. De Fazio, B. Doyon, and **F. Ravanini**, On the Hydrodynamics of Unstable Excitations, JHEP 09 (2020) 045, arXiv:2005.11266.
- O.A. Castro-Alvaredo, B. Doyon and **F. Ravanini**, Irreversibility of the renormalization group flow in non-unitary quantum field theory, J. Phys. A50 424002 (2017). Part of "John Cardy's scale-invariant journey in low dimensions: a special issue for his 70th birthday", arXiv:1706.01871.
- D. Bianchini, O.A. Castro-Alvaredo, B. Doyon, E. Levi and **F. Ravanini**, Entanglement Entropy of Non Unitary Conformal Field Theory, J. Phys A48 (2015) 4 A4FT01. This article has appeared on IoP "Highlights of 2015". arXiv:1405.2804.

Entanglement Entropy of Non-Unitary Models

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- This was interesting because in many other contexts (ie free energy of TBA) we know that the central charge c generalises to the effective central charge $c_{\text{eff}} = c - 24\Delta_{\text{min}}$ in non-unitary models where c may be 0 or negative [Itzykson, Saleur and Zuber'86].

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where p is related to the properties of the operator

$$L_0 = \Delta I + N, \quad N^r = 0, \quad 0 \leq p \leq r-1$$

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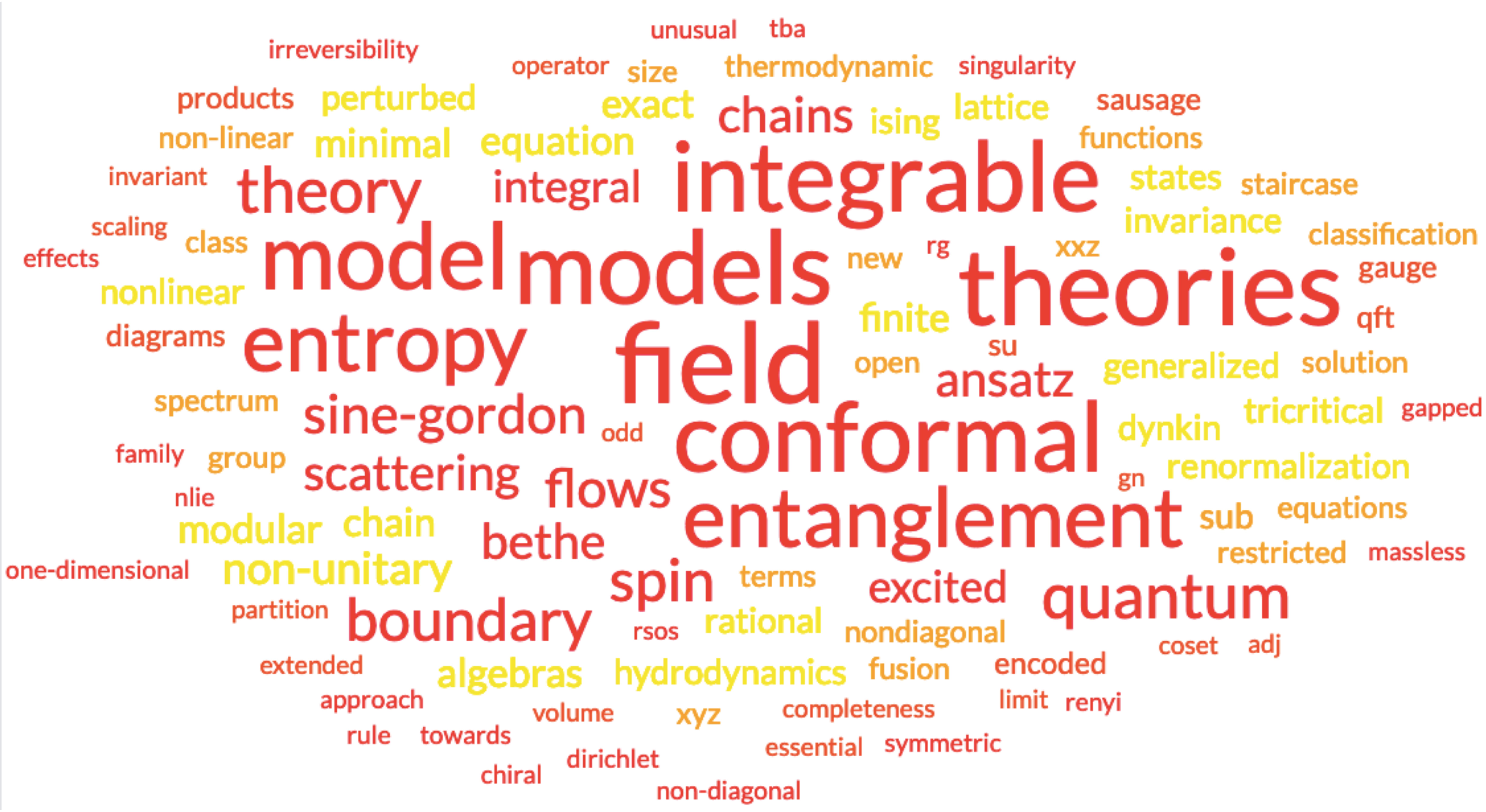
$$S_n(\ell) = \frac{\langle \mathcal{T}_\phi(0) \mathcal{T}_\phi^\dagger(\ell) \rangle}{\langle \phi(0) \phi^\dagger(\ell) \rangle^n}, \qquad \mathcal{T}_\phi \propto \lim_{x \to 0} x^{2(1-1/n)\Delta} \mathcal{T}(0) \phi(x), \qquad \Delta_c = \Delta_{\mathcal{T}} + \frac{\Delta_\phi}{n}$$

Ok but, what is he actually known for?



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tba
irreversibility
operator
size
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singularity
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perturbed
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Crescentine INFN Divina Comedia

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Italian History

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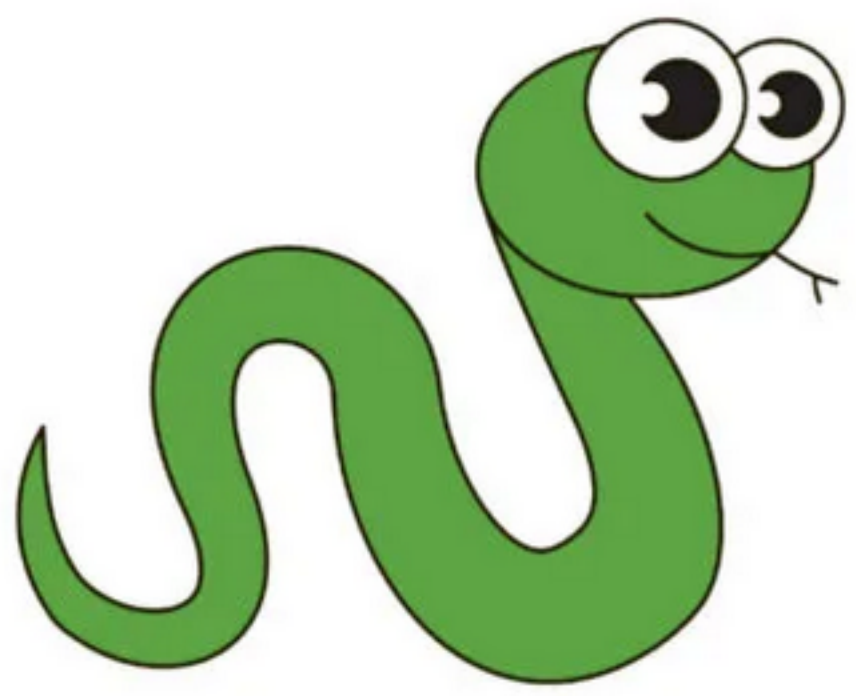

Tateo’s Snake

Food

Lamborghini

Crescentine

INFN

Divina Comedia

Italian Bureaucracy

Campanilismo

Italian History

Romagnolo

Finally: a Dark Secret

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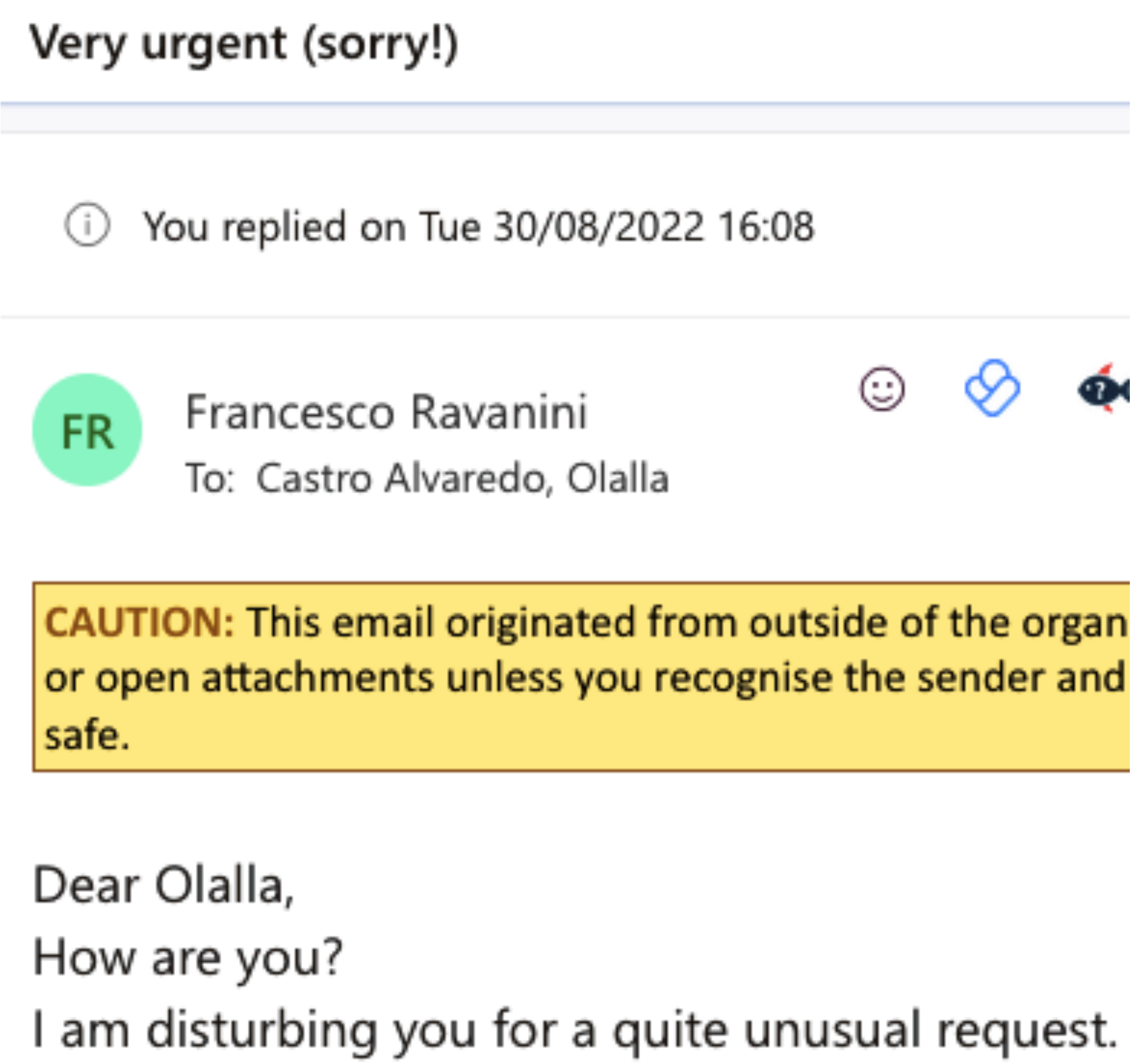

Very urgent (sorry!)

You replied on Tue 30/08/2022 16:08

FR Francesco Ravanini
To: Castro Alvaredo, Olalla

CAUTION: This email originated from outside of the organ
or open attachments unless you recognise the sender and
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Dear Olalla,
How are you?
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Wed 21/06/2023 10:40

CAUTION: This email originated from outside of the organisation. Do not click links or open attachments unless you recognise the sender and believe the content to be safe.

Dear Olalla,
I have various things to discuss with you, and it is too long to write all by email.
Can we have a video-call today? I propose at 5 p.m. Italian time (4 p.m. UK). Skype call should be enough.

2. The very urgent and cryptic email

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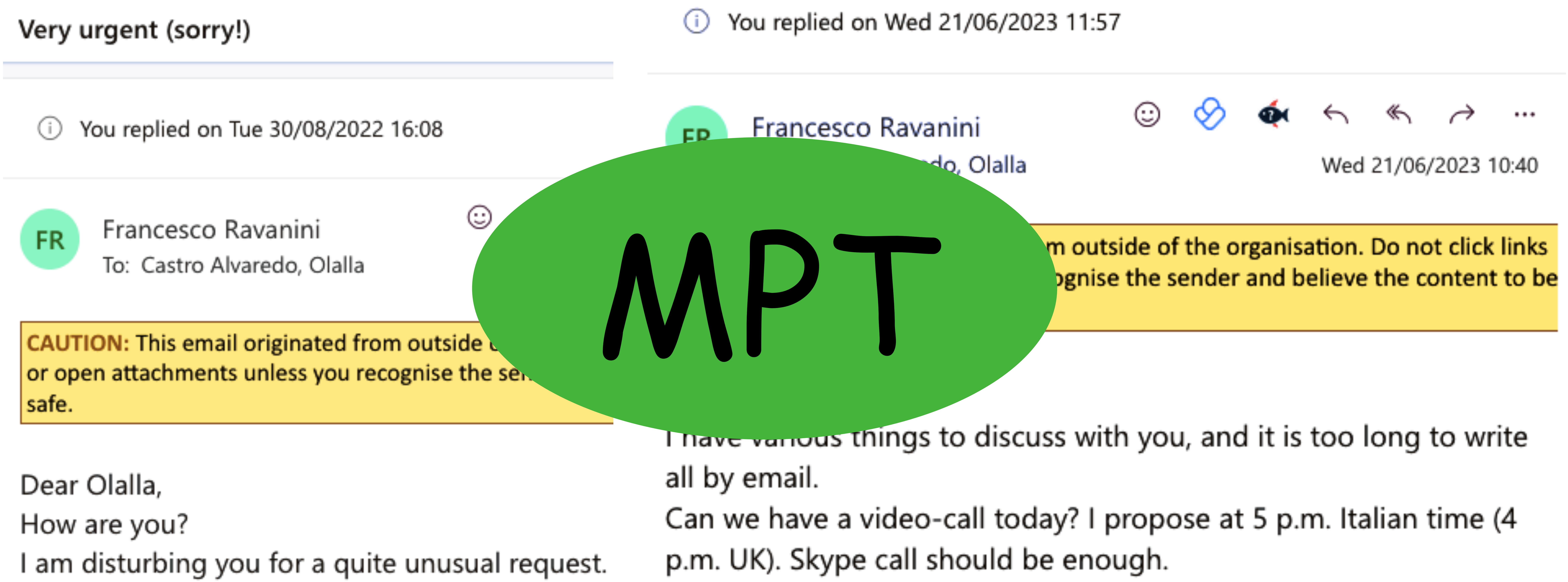

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And this is How I Learned To Stop Worrying And Love Francesco

Minimal Form Factors: Sudently and Unexpectedly Something New, Interesting, and Useful to Say

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Work in preparation with Stefano Negro (to appear soon)

Postdoctoral Research Associate at New York University

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This work is heavily inspired by the work Fabio spoke about in the previous talk!

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- We know however that an S-matrix such as that of the sinh-Gordon model is also a CDD factor.

$$S(\theta) = \frac{\tanh\frac{1}{2}\left(\theta - \frac{i\pi B}{2}\right)}{\tanh\frac{1}{2}\left(\theta + \frac{i\pi B}{2}\right)}$$

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Is there a relationship between these two types of CDD factor?

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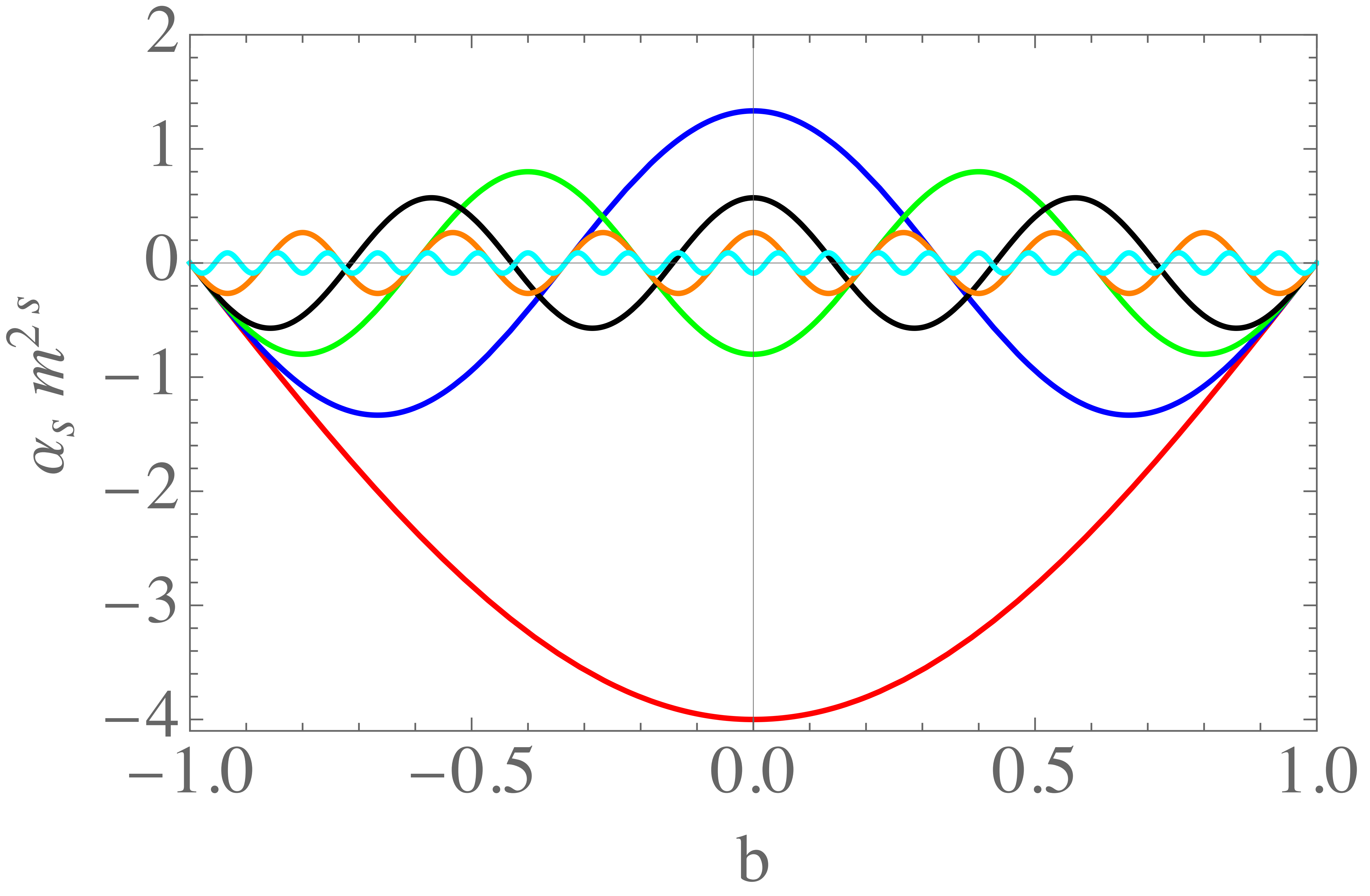

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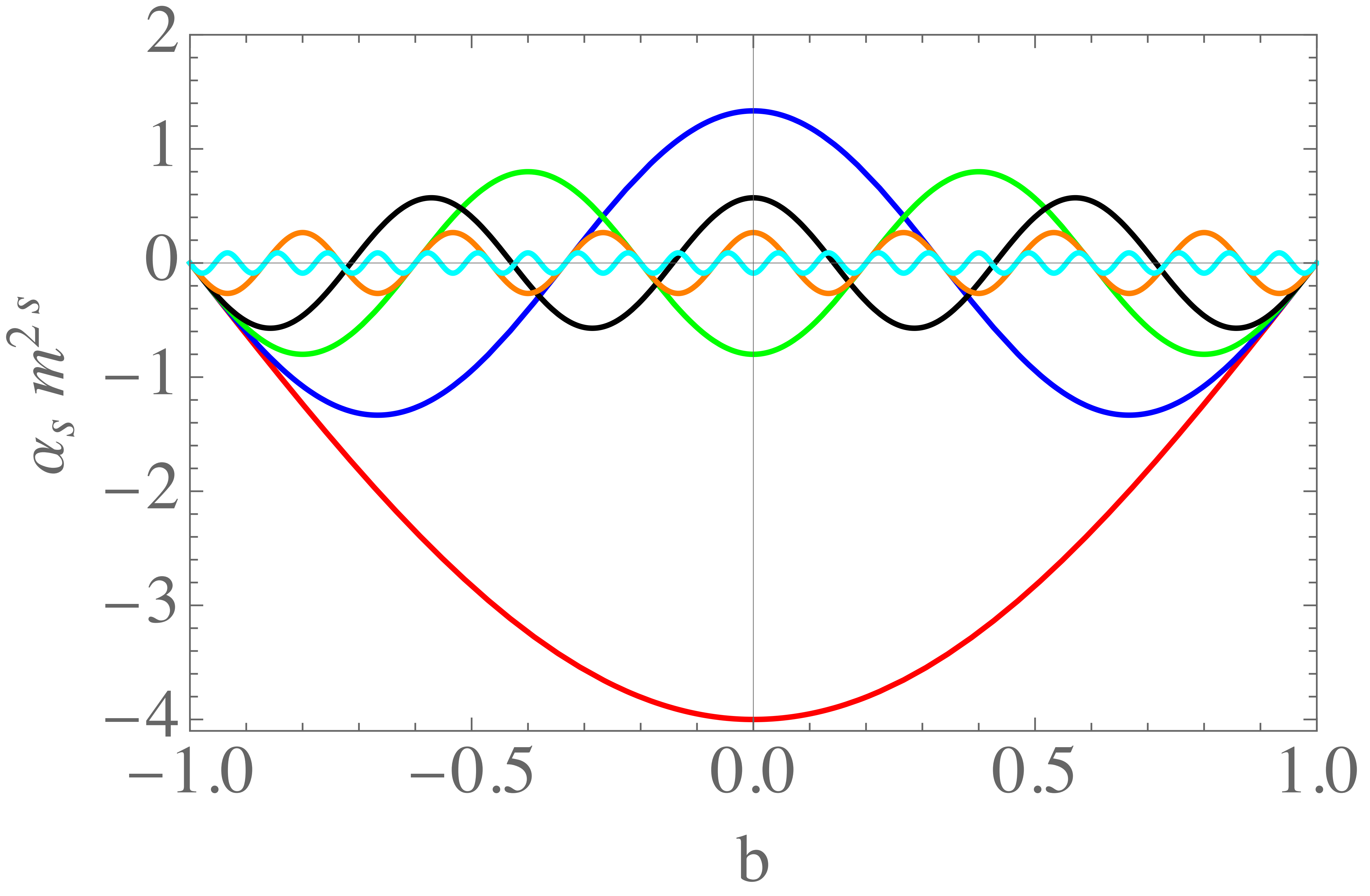

- The sinh-Gordon model can be seen as the Ising model perturbed by an infinite number of irrelevant perturbations for carefully chosen couplings [LeClair'21; Ahn & LeClair'22]

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- There are several properties of interest:

1. The solution is very simple and general. The minimal form factors gets modified by a universal multiplicative factor, just like the S-matrix!
2. The solution depends on additional parameters not present in the S-matrix. In a sense, it contains its own CDD factor

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- Since the sinh-Gordon S-matrix can be written in the "$\mathrm{T}\bar{\mathrm{T}}$-form" then it follows that the sinh-Gordon MFF should somehow admit a "$\mathrm{T}\bar{\mathrm{T}}$" representation as well.

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Minimal Form Factor Representations

- Since the sinh-Gordon S-matrix can be written in the "$T\bar{T}$-form" then it follows that the sinh-Gordon MFF should somehow admit a "$T\bar{T}$" representation as well.
- This is indeed the case:

$$F_{\min}^{\mathrm{sG}}(\theta) = F_{\min}^{\mathrm{Ising}}(\theta) e^{-\frac{\vartheta}{2\pi} i \log \Phi_{\alpha}^{\mathrm{sG}}(\theta)} C_{\beta}^{\mathrm{sG}}(\theta) \quad \text{with} \quad \vartheta = i\pi - \theta$$

- Let's have a closer look at all these functions.....

The Formula

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"Integration constant" fixed by asymptotics

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$$F_{\min}(\beta, B) = \mathcal{N}\prod_{k=0}^{N-1}\left[\frac{\left(1+\left(\frac{\hat{\beta}/2\pi}{k+\frac{1}{2}}\right)^2\right)\left(1+\left(\frac{\hat{\beta}/2\pi}{k+\frac{3}{2}-\frac{B}{4}}\right)^2\right)\left(1+\left(\frac{\hat{\beta}/2\pi}{k+1+\frac{B}{4}}\right)^2\right)}{\left(1+\left(\frac{\hat{\beta}/2\pi}{k+\frac{3}{2}}\right)^2\right)\left(1+\left(\frac{\hat{\beta}/2\pi}{k+\frac{1}{2}+\frac{B}{4}}\right)^2\right)\left(1+\left(\frac{\hat{\beta}/2\pi}{k+1-\frac{B}{4}}\right)^2\right)}\right]^{k+1}$$

$$\times \exp\left[8\int_0^\infty \frac{dx}{x}\frac{\sinh\left(\frac{xB}{4}\right)\sinh\left(\frac{x}{2}(1-\frac{B}{2})\right)\sinh\frac{x}{2}}{\sinh^2 x}(N+1-N\,e^{-2x})\,e^{-2Nx}\sin^2\left(\frac{x\hat{\beta}}{2\pi}\right)\right]$$

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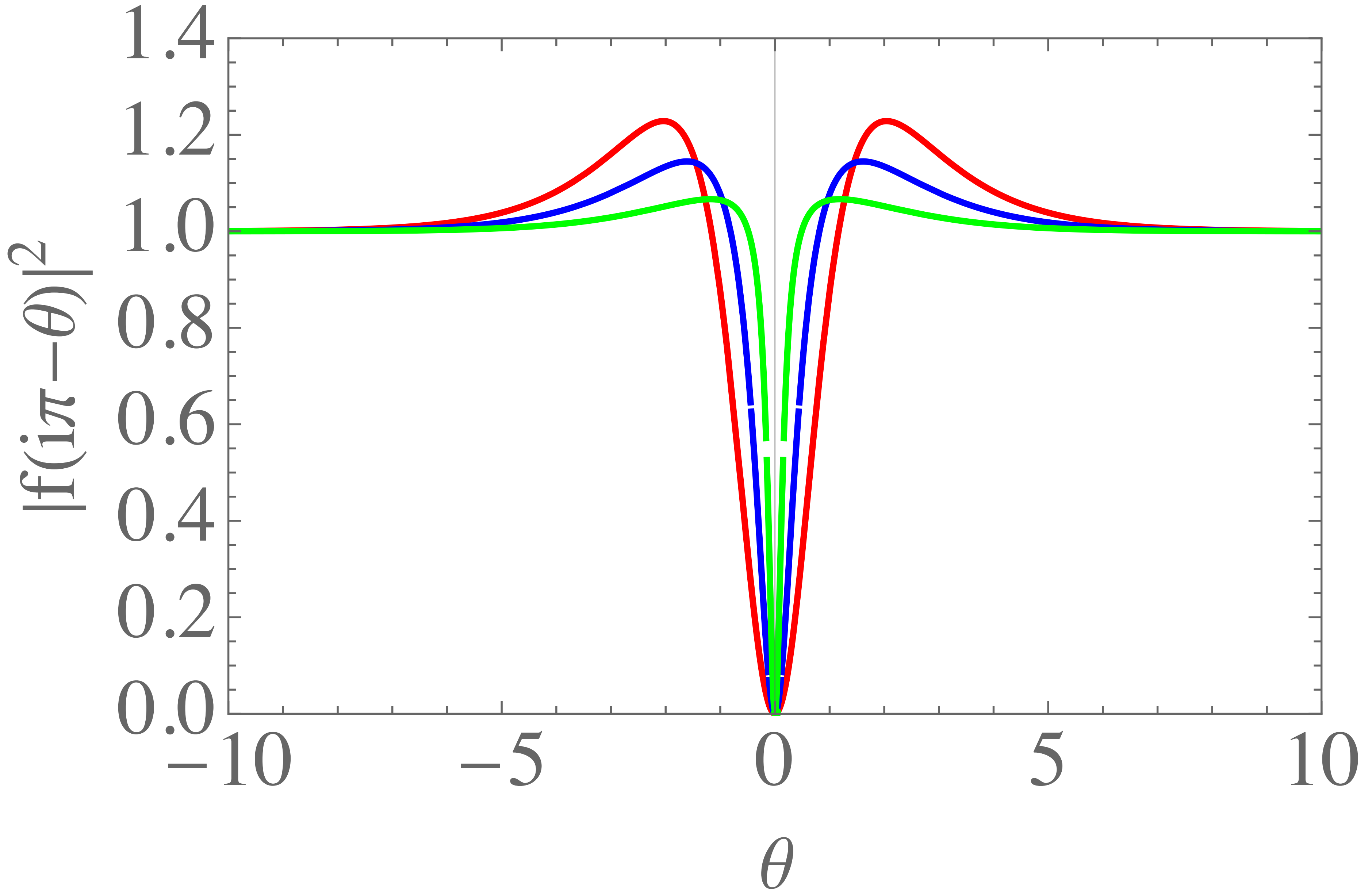

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- Having this representation for sinh-Gordon means that we effectively have it for every diagonal IQFT since the S-matrix and MFF of sinh-Gordon is a “standard block” for more complicated theories [Dorey, Exact S-Matrices’98; Mussardo, Book’10]

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- This work also proves that the MFF "CDD" factor Fabio spoke about plays a crucial role in standard theories. It actually ensures that the MFF has "desirable asymptotics" rather than the unusual asymptotics of the MFF for a single $\mathrm{T\bar{T}}$-perturbation.


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